Indian Statistical Institute, Bangalore
B. Math.
First Year, First Semester
Analysis I

Back Paper Examination — Date : Dec. 26, 2016
Maximum marks: 100 — Time: 3 hours

1. Let $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ be bounded real sequences. Show that
$$\limsup_{n\to\infty}(a_n+b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n.$$
Give an example to show that equality may not hold. [15]

2. Show that every convergent sequence is bounded but the converse is not true. [15]

3. Let a, b be real numbers with $a < b$ and let $f : [a, b] \to \mathbb{R}$ be a function. Suppose f is differentiable at $c \in [a, b]$, show that f is continuous at c. Show that the converse is not true. [15]

4. Let $g : \mathbb{R} \to \mathbb{R}$ be a function satisfying
$$|g(x) - g(y)| \leq |x - y|^{1+\beta}, \quad \forall x, y \in \mathbb{R},$$
for some $\beta > 0$. Show that g is a constant function. [15]

5. Fix $0 < t < 1$. Determine the convergence of the series $\sum_{n=1}^{\infty} b_n$, where $b_n = t^{(n+1)}$ if n is odd and $b_n = t^{n-1}$ if n is even. Show that ratio test fails but root test is applicable. [15]

6. Let $h : [0, 1] \to \mathbb{R}$ be a continuous function such that $h(0) = h(1)$. Show that there exists $0 \leq c \leq \frac{1}{2}$, such that $h(c) = h(c + \frac{1}{2})$. [15]

7. State and prove Mean Value Theorem (without assuming Rolle's theorem). [15]